Special collision dynamics of massless fermions leading to confinement

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Abstract

This is a theory for the fundamental structure of nature based on spinors as unique building blocks. Confinement of the spinors in particles is due uniquely to the dynamics of the collisions between them. The interaction operator contains only spin and momentum operators. The eigenfunctions of the interaction operator have negative as well as positive eigenvalues, limited in magnitude to the Planck energy. Interactions in one-center particles between spinors with eigenvalues of different sign tend to bring the energy of the particle closer to zero. At the origin of the energy reduction through interaction is the spin precession, taking place when two spinors of different type come together. The model can explain the existence of stable particles and all the known forces, including gravitation.

introduction

The rules of wave mechanics demand large wavenumbers for particles which are confined in small volumes. Particles like the electron or the quarks, which are smaller than any measurable size, should therefore have a much larger rest energy than they actually have. In the existing theories up to now, fields are introduced to provide negative potential energy. To make that work, an artificial distinction has to be made between real particles and virtual field-carrying particles. The particles of the fields are allowed to disobey the rules which they are supposed to save. This is an unsatisfactory situation.

We will show that the rules can be saved in a different way, without the artificial introduction of fields. To meet this goal we introduce an interaction operator based only on spin-and momentum operators, and let it operate on two-vectors. Accepting the eigenstates of this operator which have negative as well as positive eigenvalues, we arrive at two different types of spinors with collision dynamics which lead to confinement. Accepting the spinors with negative eigenvalues as real does not lead to a radiation catastrophe because radiation brings the energy of the emitting particle to zero. The energy ladder of a one-center particle does

not extend to minus infinity but is limited in magnitude by the Planck energy. All the known particles are formed by local interactions between the spinors which constitute them.

This gives us surprising results. For example the result that gravitation is just another manifestation of electromagnetism.

1 The fundamental Hamiltonian and some commutation relations.

We will base our construction of physical reality on the following interaction operator H, which we will call an Hamiltonian, although it can not in general be diagonalised when acting on localized particles :

$$H = c(\hat{\sigma}.\vec{p}) \tag{1}$$

where c is the velocity of light, $\hat{\sigma}$ is a vector representation of Pauli spin matrices and $\vec{p}$ is the momentum operator $-i\hbar\vec{\nabla} = -i\hbar\vec{grad}$.

The states on which H operates are two-vectors ψ:

$$\psi = \left(\begin{array}{c} f(\vec{r},t) \\ g(\vec{r},t) \end{array} \right) \tag{2}$$

We will leave the time dependence out of the functions further on because that can be deduced from motions on the light cone, putting $e^{i\omega t}$ equal to e^{ickt} .

Plane wave spinor eigenfunctions of H are ψ_+ and ψ_-:

$$\psi_+ = \left(\begin{array}{c} N \exp i\vec{k}.\vec{r} \\ 0 \end{array} \right) \qquad \psi_- = \left(\begin{array}{c} 0 \\ N \exp i\vec{k}.\vec{r} \end{array} \right) \tag{3}$$

Where N is a normalisation constant and spins are quantised in the direction of $\vec{k}$. The eigenvalues are $+\hbar k$ and $-\hbar k$ respectively.
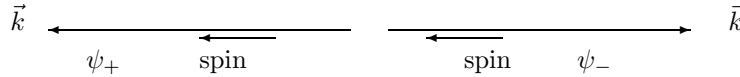
H acts exclusively on sums of spinors which move with the speed of light. This can be concluded by determining the velocity operator $\frac{d\vec{r}}{dt}$:

$$\frac{d\vec{r}}{dt} = \frac{i}{\hbar}[\vec{r}, H] = c\hat{\sigma} \tag{4}$$

The motion of fermions with the velocity of light was recognized long ago, soon after the introduction of the theory of Dirac for relativistic fermions. It was called "Zitterbewegung" [1] ,was interpreted as an irregular motion due to uncertainty, and was then neglected. It is a central point in this paper.

It is important to note that the motion of a spinor is along the spin vector, not along the momentum vector. The motion of the center of mass, however, is along the momentum vector, as it should be. This is deduced from the definition

of the position vector $\vec{r}_M$ of the center of mass and from the commutation of $\vec{r}_M$ with H:

$$\vec{r}_M = \frac{\vec{r}H}{<H>} \tag{5}$$

This definition is the operator form of the classical formula $\dfrac{\sum \vec{r}_i m_i}{\sum m_i}$.

$<H>$ is the expectation value of H, integrated over a limited space containing all the spinors which make up the one-center particle under consideration. The motion of the center of mass is determined by:

$$\frac{d\vec{r}_M}{dt} = \frac{i}{\hbar} \frac{[\vec{r}H, H]}{<H>} = \frac{c^2 \vec{p}}{<H>} = \frac{c^2 \vec{p}}{mc^2} = \frac{\vec{p}}{m} \tag{6}$$

We have left out a vector product of spin and momentum in this result. This is justified for non-interacting spinors, with spins quantised in the direction of $\vec{k}$. In the full expression for interacting spinors in particles the sum of the vector product terms is zero on account of the central symmetry. The result in equation (6) is as we expect from classical experience.

Commutation relations for $\hat{\sigma}$, for $\vec{p}$ and for $[(\vec{r} \times \vec{p}) + \frac{\hbar}{2}\hat{\sigma}]$ give us the following results:

$$\frac{d\hat{\sigma}}{dt} = \frac{i}{\hbar}[\hat{\sigma}, H] = \frac{2c}{\hbar}\hat{\sigma} \times \vec{p} \qquad \frac{d\vec{p}}{dt} = 0 \qquad \frac{d[(\vec{r} \times \vec{p}) + \frac{\hbar}{2}\hat{\sigma}]}{dt} = 0 \tag{7}$$

The first equation in (7) is the key equation to understand the origin of confinement without fields, as will be shown in the next section.

If $<H>$ is calculated for one isolated particle, in which all the spinors follow closed paths around one center, the result may be negative as well as positive, depending on which sign of the spin-momentum product prevails. However, we may keep the convention that $|<H>|$, the norm of $<H>$, is the energy of the particle.

The sign of $<H>$ can be identified with the sign of the electric charge.

The difference between the theory which we present here and the Dirac theory [1,2] for relativistic fermions lies mainly in the introduction of lower-level structures and in the acceptance of negative values for $<H>$. These were rejected as unphysical because of a "radiation catastrophe". In our model all energy levels are brought to zero by interactions, not to minus infinity.

The Dirac Hamiltonian [1,2] is an approximation of H, in the following way:

$$H = c\hat{\sigma}.\vec{p} = c\hat{\sigma}.\vec{p}_M + c\hat{\sigma}.(\vec{p} - \vec{p}_M) \tag{8}$$

$\vec{p}_M$ represents the collective momentum of the whole particle, deduced from the motion of the center of mass. The Dirac Hamiltonian $H_D = c\hat{\alpha}.\vec{p} + m_o c^2 \hat{\beta}$ with $\hat{\alpha}$ and $\hat{\beta}$ four-by-four matrices built out of $\hat{\sigma}$ and unit matrices is a kind of renormalised form of equation (1), with $\vec{p}$ representing the center-of-mass momentum and the rest energy $m_o c^2$ corresponding to $|<H>|$ in the center-of-mass frame. By taking this definition of $\vec{p}$, the energy-reducing interactions were lost in H_D. Also the $\hat{\sigma}$ submatrices of $\hat{\alpha}$ in the Dirac theory do not refer to the spin operator acting on spinors but rather to an averaged angular momentum operator for the whole particle.

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2 Collision dynamics

Spinors interact with one another only locally, because the interaction operator H is a local operator. As shown in equation (7), $\hat{\sigma}$ rotates in precession around $\vec{p}$, much like a magnetic moment precesses around a magnetic field. As long as the spinors are separated in space-time, the spins are aligned with the momenta. When spinors come closer than a minimum distance r_o, defined further on, the spins and the momenta are added up and the total spin will rotate around the total momentum.

The total momentum and also $\hat{\sigma}^2$ are conserved in the collisions. Collisions between spinors with charges of the same sign are fairly classical. The outgoing vectors of spin and momentum are simply rotated with respect to the incoming vectors, rotated around the total momentum.

Collisions of spinors with different charge signs, however, are very special. They have no similarity to what we know in the macroscopic world. They are at the origin of confinement in localized particles. The spin precession produces here a kind of bending of the outgoing tracks towards one another, or, if we keep all the vectors in one plane, a reflection of the motion on the line of the total momentum. To see that we must remember that the direction of motion is along the spins. Two examples are shown in figure (1):
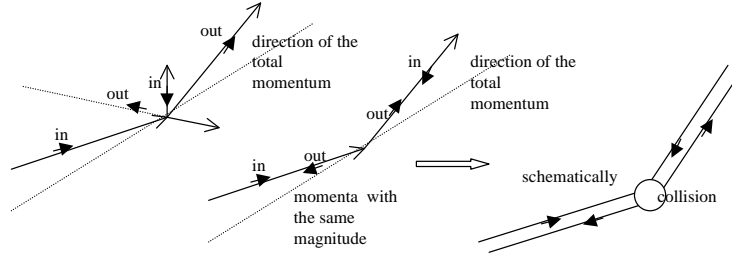

figure 1: collisions of fermions with opposite charge signs

On tracks bent by this kind of collisions, four spinors can run around the sides of a square and many intertwined spinors can form circular paths.

There is no need to introduce a field to bend the tracks. The field is the other spinor. Because spinors with different charges stay together and are transformed into each other, the energy of a particle is reduced as a consequence of periodical switching between positive and negative levels.

3 The photon

A neutral combination of fermionic spinors with cylindrical symmetry has all the properties of the particles which we know as photons. In a photon, the total

angular momentum with respect to the axis of the cylinder, which is the axis of propagation, is constant. It is composed of a spin part and an orbital part, both having half integer quantum numbers. Cylindrical spinor functions for a particle which is stable under the interaction operator H must be eigenfunctions of H^2, i.e. of the $\vec{\nabla}^2$ operator.

We can read in handbooks on mathematical functions that the eigenfunctions of the $\vec{\nabla}^2$ operator which are finite on the axis are the cylindrical Bessel functions. In cylindrical coordinates with the axis of the cylinder along z, the radius r and the azimuthal angle φ are defined in the xy plane and the lowest order cylindrical Bessel functions are:

$$u_+(z,r,\varphi) = Ne^{ikz}\frac{sin(k_o r)}{\sqrt{k_o r}}e^{i\frac{\varphi}{2}} \qquad v_+(z,r,\varphi) = Ne^{ikz}\left(\frac{\cos(k_o r)}{\sqrt{k_o r}} - \frac{\sin(k_o r)}{\sqrt{(k_o r)^3}}\right)e^{3i\frac{\varphi}{2}}$$
$$(9)$$

and

$$u_-(z,r,\varphi) = Ne^{ikz}\left(\frac{\cos(k_o r)}{\sqrt{k_o r}} - \frac{sin(k_o r)}{\sqrt{(k_o r)^3}}\right)e^{-3i\frac{\varphi}{2}} \qquad v_-(z,r,\varphi) = Ne^{ikz}\frac{sin(k_o r)}{\sqrt{k_o r}}e^{-i\frac{\varphi}{2}}$$
$$(10)$$

The parameter k_o in these functions must be chosen to match the smallest meaningful distance r_o so that $k_o r_o = 1$. We will take the Planck length [3] $\simeq 10^{-35}m$ for r_o.

We can see that the following ψ_{ph}, with spin quantisation in the z-direction, is an eigenfunction of H^2 with all the properties of the phonon:

$$\psi_\pm = \left(\begin{array}{c} u_\pm \\ v_\pm \end{array} \right) \qquad (11)$$

This spinor has standing waves in the radial direction which provide the necessary communication between the circular currents around the axis of the cylinder. It can transport at the same time a positive contribution to the energy of a detector and a negative contribution to the energy of a source, transmitting also an angular momentum $\hbar$. Detector and source are on the same footing; no "strange actions at a distance". With k equal to zero we have a vacuum photon, which carries no energy but can couple particles together.

We can depict the photon as a sort of hollow tube, with zero density on the axis, as shown in figure (2)
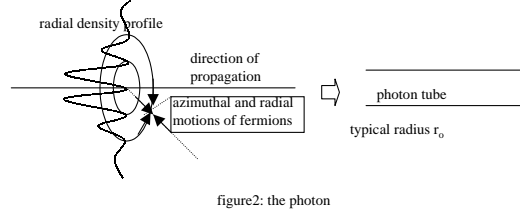
figure2: the photon

When the photon is transmitting "force" between two charged particles in a stationary bound state, it can be imagined as a whirl with the axis in the most favorable direction for the binding, that is perpendicular to the line connecting the particles.

4 Localised particles; the electron

From here on the systems become too complicated for complete analytical description. We will only be able to give rough pictures, which will have to be refined with the aid of large computers.

A spinor for one type of charge can not be radially confined without reflections caused by the interaction with photons. The most efficient mode to achieve radial confinement is probably the internal reflection of a charged spinor moving axially in the inside of a photon,as shown in figure (3)
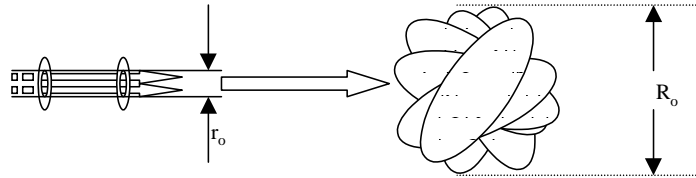

figure 3: a charged photon tube curled up to form a particle

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We will call this a charged photon tube. We can imagine internal reflections to prevail over outward scattering by correlated radial motions of the charged interior of the tube and of the components of the photon. The photon shield will probably pick up energy in that process but reduce the energy of the whole system.

Due to the interaction between the charged spinor and the photon the cylindrical symmetry may be broken spontaneously. The charged photon tube curls up and the cylinder symmetry survives only locally. The tube may now close on itself and if the conditions are right for self-consistency we may have a confined fermion. It resembles a ball of yarn as shown in figure (3).

The ball can be hollow to prevent the formation of too high momentum components in the center.

The total length of the woven charged tube must fit the condition that the path integral of the phase gradient is equal to an integer times 2π. Although the exact determination of the structure of this ball will be a formidable task, we may try to guess at the relation between the radius R of the ball and its energy. With $k_o r_o = 1$ and r_o the Planck length [3] of $10^{-35}m$, $\hbar k_o$ is of the order of 10Nm. The energy of one elementary fermion circulating at a distance r_o from a center would then be of the order of $10^9 J$. The energy of a curled-up ball will be inversely proportional to the number of Planck-size cells in the volume occupied by the charged current, each cell being visited only once in a cycle to avoid self-intersection. If the ball has a radius R and no hole, the number of Planck-size cells is of the order of $(\frac{R}{r_o})^3$, reducing the momentum of the fermion by that factor.

The rest energy of the electron is $8.10^{-14}J$. Putting this equal to $\frac{10^9.(r_o)^3}{R^3}$ we find R equal to approximately $10^{-28}m$ or 10^7 Planck lengths. In this estimation we have completely neglected the contribution of the photon shield to the energy of the particle. The more compensation given by the photon shield, the higher the bare momentum of the charged spinor and the lower the radius of the particle. On the other hand the radius of the electron may be bigger if there is a central hole. This would not necessarily show up in collisions with other particles because the hole would make the ball softer.

Particles like the quarks, with an energy which is rather close to the energy of the electron on the Planck scale, both being relatively very small, can be similar to the electron.

5 The interactions

The interactions between particles is mediated by photons. In a simplified scheme we take two different particles with momenta p_{1p} and p_{2p} respectively, with the centers at a distance r from one another. A photon whirl makes a connection between the two and can take up momentum components simultaneously from both currents, as shown in figure (4).
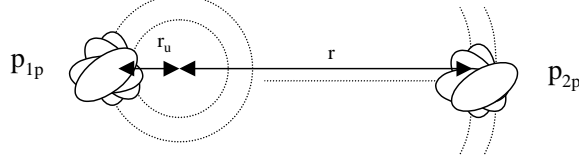
figure 4: the connection between two particles, with momentum p_{1p} and p_{2p} respectively, via a photon or photons. The distance r_u is adapted to make the momentum of the photon at that distance from the center, $p_o r_o / r_u$ equal to the momentum which is switched in the particle. The momentum transfer to the second particle is reduced by r_u / r.

The system will radiate out energy if it is not in a stable state. In a stable situation, in which every spinor in the system returns periodically to a given initial condition, the photon whirl receives simultaneously from both particles equal and opposite spinor components which it takes up without changing its energy. Each of the particles is subjected, via collisions with the photon whirl, to a reduction of its energy with respect to the energy it would have without the interaction. This reduction can be estimated by considering a match of the momentum vectors of the photon with the momentum of the particle on one side, say p_{1p}. To make this match we must consider the bare momentum p_u of the spinor unit with which the photon interacts. For a charged particle like the electron this is the bare momentum of the central spinor. For interaction between bosons which carry energy it is the bare spinor momentum $p_o = \hbar k_o$. If the photon is positioned at a distance $r_u = \frac{\hbar}{p_u}$ from the first particle, in a way to get maximum contact interaction, it can contribute a fraction $\frac{r_u}{r}$ at the position of the second particle to reduce the energy of the system. Putting this together in a formula for the interactive reduction of energy we arrive at:

$$interactive\ \ energy\ \ reduction\ \ = c\frac{p_{1p}p_{2p}}{p_u}\frac{r_u}{r} = c\hbar\frac{p_{1p}p_{2p}}{(p_u^2)r} \qquad (12)$$

with cp_{1p} and cp_{2p} the energies of the interacting particles when they are isolated, $p_u = \frac{\hbar}{r_u}$ the momentum of the bare spinor unit with which the photon couples and r the distance between the particles.

If we apply this perturbation energy correction to charged particles, p_{1p} and p_{2p} are lower than p_u by the amount of compensating momentum in the photon shields of the particles.

We know from the value of the fine structure constant $\frac{1}{137}$ that by putting

$$electrical\ \ interaction\ \ \frac{e^2}{4\pi\epsilon_o r} = c\hbar\frac{p_{1p}p_{2p}}{(p_u^2)r} \qquad (13)$$

the reduction factor $\frac{p_{1p}p_{2p}}{(p_u^2)}$ must be $\frac{1}{137}$.

Gravitation is an interaction between bosons. In this interaction the mediating photons couple two-by-two the elementary spinors which are not completely cancelling each others energy in the particles. Here we must take the bare

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$p_o = \hbar k_o$ momentum as the unit for interaction with the photons. Taking for the momenta which must be compensated $m_1 c$ and $m_2 c$, deduced from the energies of the particles, we arrive at:

$$gravitational \;\; interaction \;\; \frac{Gm_1m_2}{r} = c\hbar \frac{cm_1cm_2}{(p_o)^2 r} = c\frac{cm_1cm_2(r_o)^2}{\hbar r} \qquad (14)$$

and this leads to the value of the Planck length when the known value of the gravitational constant G is inserted.

The interaction of particles can also give short-range forces, like the strong and the weak nuclear force. The short range of those forces points towards a rupture of the bond between particles. The obvious candidate for these forces is a connection via charged photon tubes, entangled into the connected particles. A particle like an electron , with a total length for the charged photon tube of the order of $10^{-14}m$, partially unwound and hooked into similar particles, gives us the right order of magnitude for the range of the nuclear forces.

References

[1] treated in most textbooks on relativistic quantum mechanics e.g. W.Greiner, Relativistic quantum mechanics; Springer Verlag 1990 ISBN3-540-50986-0

[2] P.A.M.Dirac, Proc.Roy.Soc.A117,610(1928);A118;351(1928) "The Principles of Quantum Mechanics"

[3] original reference: M.Planck, Sitz.Preuss.Acad.d.Wissensch., Berlin 1899, pp.440-470
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